A Brief Story of Muon Collider

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1 Introduction and Motivation of Muon collider

Traditional colliders face various constraints and inefficiencies that limit their potential in exploring the frontiers of particle physics. For instance, circular e^+e^- colliders, which accelerate electron-positron beams, encounter significant challenges at high energies due to extensive synchrotron radiation. This phenomenon, where accelerated charged particles lose energy through radiation, becomes more pronounced as the energy of the particles increases, making it less efficient for probing the higher reaches of energy scales. On the other hand, hadron colliders like the Large Hadron Collider (LHC) accelerate composite particles, such as proton beams. The interactions at these colliders occur at the parton level (quarks and gluons inside the protons), which only utilize a fraction of the total energy available in the beams. This leads to a considerable amount of energy waste and complicates efforts to probe higher energy scales directly.

Muons, as fundamental leptons, offer a compelling solution to these challenges. With its large mass compared to electron, muons significantly mitigate the problem of synchrotron radiation that plagues e^+e^- colliders. Additionally, being fundamental particles, muons use their entire energy in collisions, unlike the partonic interactions in hadron colliders. This makes muons exceptionally advantageous for a next-generation particle collider, promising more efficient and effective probing of uncharted high-energy territories.

1.1 Energy efficiency

Synchrotron radiation As the second generation lepton, muon ($m_\mu \simeq 106$ MeV) is roughly 200 times the mass of the electron ($m_e \simeq 0.511$ MeV). Suppose a circular accelerator, the synchrotron radiation power of the high-energy particle is given by [1]

$$P_\gamma = \frac{q^2}{6\pi\epsilon_0 c^3}\left(\frac{c^2}{R}\right)\left(\frac{E}{mc^2}\right)^4. \tag{1}$$

$\mu^-.e^-$ has the same absolute charge. For the same energy E, the ratio of synchrotron radiation of two leptons are inversely proportional to mass to the forth power. Therefore $P_\gamma^\mu/P_\gamma^e \sim 10^{-9}$. The radiation power from muon collider is 9 orders of magnitude less than electron colliders.

PDF advantage For a direct μ^+, μ^- collision event at center of mass energy $E_{\mathrm{CM}} = \sqrt{s}$, all of that energy are useful for probing high energy physics and a naive s-channel pair production estimation gives the direct reach of particles with $M \sim \sqrt{s}/2$. In pp collision events, the total cross section is given by [2]

$$\sigma(pp \to F + X) = \int_{\tau_0}^1 d\tau \sum_{ij} \frac{dL_{ij}}{d\tau}\hat{\sigma}(ij \to F + X), \tag{2}$$

where L_{ij} is the parton luminocity that is given by

$$\frac{dL_{ij}}{d\tau} = \int_0^1 dx_i dx_j f_i(x_i, Q^2) f_j(x_j, Q^2)\delta(x_i x_j - \tau). \tag{3}$$

Here $\{i, j\}$ are partons inside protons and $f_i(x_i, Q^2), f_j(x_j, Q^2)$ are there parton distribution function (PDF) with energy fraction x_i, x_j and hard energy transfer scale Q^2. $\bar{\sigma}(ij \to F + X)$ is the parton level cross section of the production channel. Summing over $\{i, j\}$ of all possible channels. $\tau \equiv \hat{s}/s$

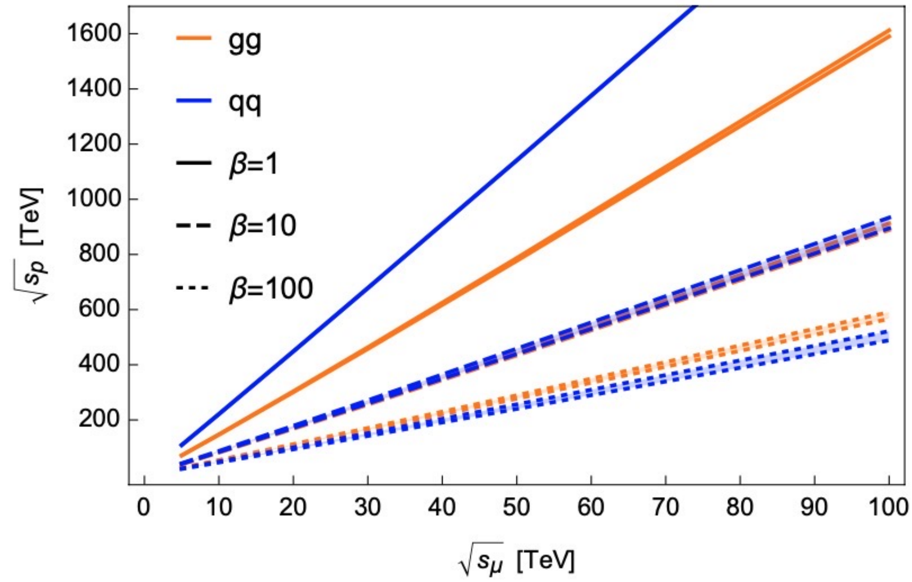

Figure 1: The COM energy for a proton collider $\sqrt{s_p}$ and a muon collider $\sqrt{s_\mu}$ such that the cross sections are the same. Characterized by β. Bands given by different choices of PDFs. [2]

is the ratio of parton proton s-variable, resemble the fraction of energy useful of a specific parton process.

For different parton process, we can define $\beta \equiv \hat{\sigma}_p/\sigma_\mu$, which accounts for the ratio of a parton cross section with the muon cross section. It differs from parton to parton. For a generic process with $\beta \sim 1$, we gain a factor of 20 in saving of the center of mass energy of muon collider compared to pp collider. For $\beta \sim 100$, muon collider only needs approximately one quarter of the energy required as well.

2 Physics cases on muon collider

2.1 Higgs physics

The discovery of the Higgs boson in 2012 marked a monumental triumph for the Higgs mechanism and the Standard Model, while simultaneously introducing a new fundamental question: why is the Higgs potential configured as we currently believe? The configuration of the Higgs potential is intricately linked to the naturalness puzzle, or the hierarchy problem, which stems from efforts to understand the underlying reasons for electroweak symmetry breaking. Precise measurements of the Higgs self-couplings and its interactions with other Standard Model particles are critical to addressing these questions. These measurements have emerged as some of the most compelling objectives for the next generation of particle colliders.

In the realm of fundamental physics at the energy frontier, the type of particle being collided does not inherently limit the scope of Higgs physics. Thus, the Higgs exploration potential of muon colliders aligns closely with that of e^+e^- colliders, yet with a significantly broader energy reach. This enhanced capability allows for a deeper investigation into Higgs phenomena. Here, we briefly discuss two different Higgs production channels and their respective cross sections, showcasing the potential of muon colliders to excel in Higgs physics by leveraging their higher energy capacities. [3]

s-Channel production The s-channel lepton production of Higgs is given by $\mu^+\mu^-$ annihilate into an off-shell Z^* and then decay into an on-shell Z and h. The cross section of this process is given by [4]:

$$\sigma(\mu^+\mu^- \to Zh) \sim \frac{G_F^2 m_Z^4}{96\pi}\frac{1}{s}F\Big(s-(m_h+m_Z)^2\Big) \tag{4}$$

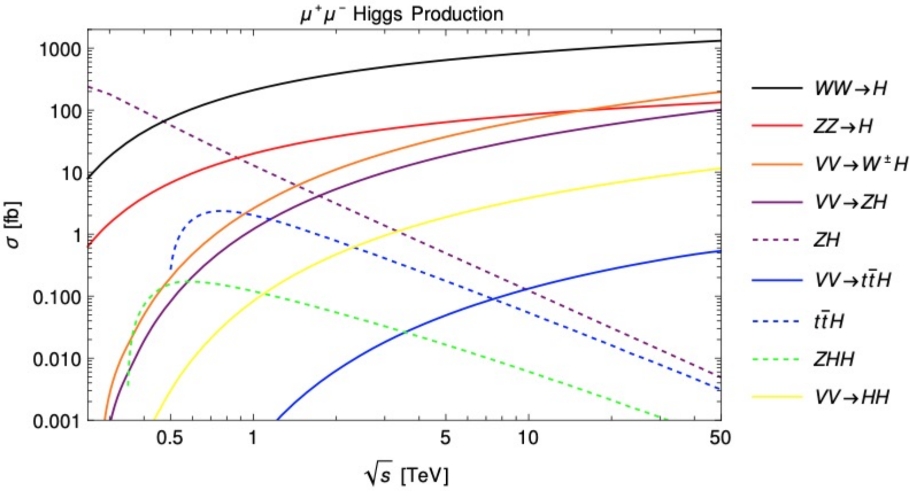

Figure 2: Cross section of various Higgs production mechanism. The dashed curves correspond s-channel annihilation production, the solid curves correspond VBF process [3]

It is proportional to $1/s$ times a function that peaks around the mass shell of final particle states $F(s-(m_Z+m_h)^2)$. The cross section grows until around 200 GeV and then decreases, which makes it less and less dominate in higher energy runs.

The other interesting channel of producing Higgs is through Vector Boson Fusion (VBF). The cross section of VBF is given by [4]:

$$\sigma(\mu^+\mu^- \to h\nu\bar{\nu}) \sim \frac{G_F^3 m_W^4}{4\sqrt{2}\pi^3}\left(\log\frac{s}{m_h^2} + \text{finite.}\right) \tag{5}$$

It grows logarithmic with COM s, makes it dominates over other processes after a threshold energy at around 350 GeV. For a high energy muon collider running at 10 TeV, which current ICMM envisioned with a possible staging of 3 TeV as a leverage. VBF processes are much more important than s-Channel production processes. However, if we decided to built a 125 GeV muon collider as the first step, in order to hit the resonance of Higgs, the s-Channel production should also be taken into consideration. Although a 125 GeV muon collider plan would be hard to compete with current e^+e^- higgs factory instead.

With the clean environment of muon colliders, the precision measurements of Higgs couplings could do much better than HL-LHC which is proposed as the near future hadron collider. Figure.(3) shows a simulated increment in precision with muon colliders of different staging possibilities. In general, we can decrease the error bar for one or two orders of magnitude of various Higgs couplings.

3 The Evolution of the Muon Collider

The idea of a muon collider dates back approximately 60 years, first surfacing in the literature of the 1960s, where the potential of muon storage rings was contemplated. The concept of a muon collider, along with the crucial requirement for muon cooling, was further developed from the 1970s through the 1990s. In a significant step forward, the Neutrino Factory and Muon Collider Collaboration (NFMCC) was established in 2000, initiating a robust research and development program focused on both the Neutrino Factory (NF) and the Muon Collider (MC).

Following the operational commencement of the Large Hadron Collider (LHC) and before the discovery of the Higgs boson, the Muon Accelerator Program (MAP) [6] was formed in 2011 as a continuation of the NFMCC. Its aim was to assess the technological feasibility of these ambitious projects. More recently, in 2019, CERN spearheaded the formation of a new International Muon Collider Collaboration (IMCC) [5] to evaluate the feasibility of constructing a high-energy muon collider and to develop an R & D program to tackle the critical challenges involved. The near-term goal for the next five years under IMCC is to determine whether investing in a conceptual design report and a demonstration program for the muon collider is scientifically justifiable.

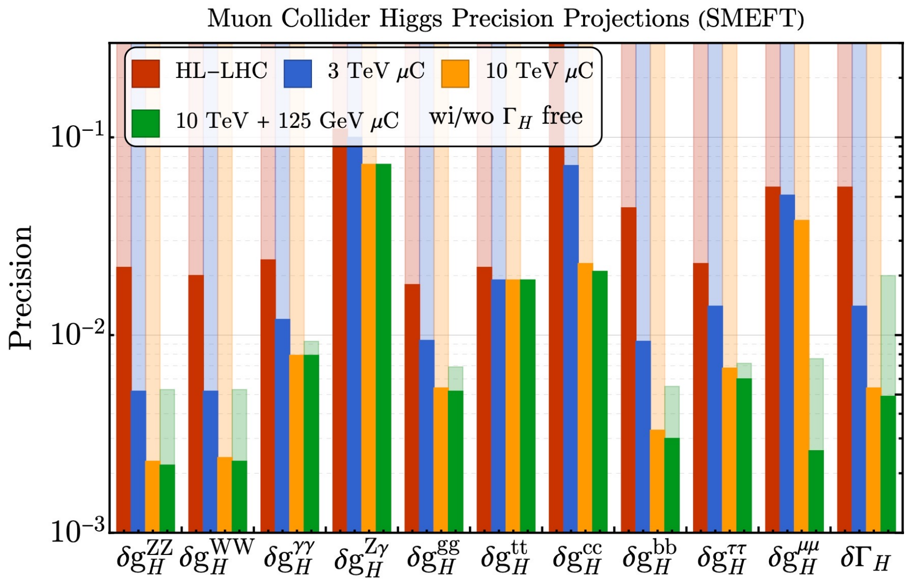

Figure 3: The one sigma precision reach on the effective Higgs couplings from a global fit of the Higgs and electroweak measurement in the SMEFT [5]

Parameter	Unit	Higgs Factory	3 TeV	10 TeV
COM Beam Energy	TeV	0.126	3	10
Collider Ring Circumference	km	0.3	4.5	10
Interaction Regions		1	2	2
Est. Integ. Luminosity	$ab^{-}1/year$	0.002	0.4	4
Peak Luminosity	10^{34} cm^{-2} s^{-1}	0.01	1.8	20
Repetition rate	Hz	15	5	5
Time between collisions	μs	1	15	33
Bunch length, rms	mm	63	5	1.5
IP beam size σ^*, rms	μm	75	3	0.9
Emittance (trans), rms	mm-mrad	200	25	25
β function at IP	cm	1.7	0.5	0.15
RF Frequency	MHz	325/1300	325/1300	325/1300
Bunches per beam		1	1	1
Plug power	MW	~ 200	~ 230	~ 300

Figure 4: A summary of parameters for the primary muon collider options considered by muon collider forum report [5]

Currently, IMCC proposes a two-staged approach: the first stage operating at a center-of-mass energy of 3 TeV, and the second stage escalating to 10 TeV. The design strategy employed by IMCC draws heavily from the groundwork laid by the MAP collaboration. This design pipeline begins with multi-megawatt proton beams striking a target to produce a copious amount of pions, which subsequently decay into μ^+ and μ^-. These muons are then subjected to a cooling process to achieve the necessary emittances for targeted luminosities. Following this, the muons are accelerated to the required collision energy and injected into a collider ring with two interaction points, setting the stage for high-energy collisions and groundbreaking scientific exploration

3.1 Technological Feasibility

Building a muon collider presents a host of scientific and engineering challenges that are currently at the forefront of the muon collider community's efforts. In this section, we will outline some of the most pressing obstacles that capture the keen interest and intensive focus of researchers and engineers involved in this groundbreaking project. These challenges are not only critical to the success of the muon collider but also offer exciting opportunities for innovation and advancement in the field of particle physics.

3.2 High tolerance target

The development of an effective target system for a muon collider is a critical engineering challenge, particularly due to the high thermal stresses induced by proton beam impacts. Current neutrino target technology has matured to sustain operations at the 1-megawatt level; however, this capacity falls significantly short of the requirements for a muon collider, which demands handling capabilities several orders of magnitude greater. Since 2012, the international collaboration known as Radiation Damage In Accelerator Target Environments (RaDIATE) has been actively engaged in research and development focused on target materials. This collaboration aims to innovate and enhance target technology to meet the rigorous demands of muon collider operations, addressing the significant gap between current capabilities and future needs. [7]

3.3 Muon Beam Cooling

The production of muons from pion decay results in a beam occupying a relatively large phase space volume, necessitating significant compression to meet the emittance requirements for high-luminosity collisions. To address this, the Muon Accelerator Program (MAP) proposed a sophisticated ionization cooling scheme [8], which is segmented into three distinct steps. Ionization cooling involves the passage of particles through a material medium where they lose energy via ionization interactions. Subsequently, the muons are directed to radio frequency (RF) cavities for re-acceleration. While momentum loss occurs in all directions, re-acceleration specifically restores only the longitudinal component, effectively reducing the net transverse momentum. This process counters the random scattering events that otherwise increase beam divergence. While the underlying theory of ionization cooling is relatively straightforward, it demands comprehensive feasibility studies and extensive numerical simulations to validate its practical application.

3.4 RF Cavities and Stability Challenges

A further critical challenge is the stability of RF cavities used in the cooling and re-acceleration stages. Given the short lifespan of muons, these processes must operate at high frequencies to be effective. The RF cavities are required to function within a strong magnetic field, which significantly complicates their operation. Experimental investigations at the MuCool Test Area at Fermilab have demonstrated that intense external magnetic fields increase the likelihood of RF breakdown events. This breakdown is primarily due to field emission electrons being bent into beamlets that can cause severe damage to the cavity surfaces, leading to eventual failure. [9]

To mitigate these issues, two types of RF cavities have been proposed by MAP: (A) Vacuum RF Cavities, (B) High-Pressure Gas-Filled Cavities. Both solutions present unique advantages and challenges, and selecting the optimal approach will require further experimental validation and development.

3.5 Neutrino flux background

One of the most pressing concerns in the operation of a muon collider is managing the substantial radiation loads and particle backgrounds [10], particularly from the decay products of ultra-high-energy muons. Muons decay into high-energy neutrinos, which, due to their energetic and highly directional nature, can interact with materials even at considerable distances from the collider. The spread angle of the neutrinos from the muon beam direction can be described by the equation:

$$\theta_\nu = \frac{1}{\gamma_\mu} = \frac{m_\mu}{E_\mu} \simeq \frac{10^{-4}}{E_\mu}[\text{TeV}] \tag{6}$$

This equation shows that the neutrinos are emitted in a narrow beam, closely aligned with the original muon trajectory. Despite their low interaction cross-section, these neutrinos can travel in almost straight lines and create charged particles along their path, leading to potential radiation hazards including significant energy deposits in biological tissues, raising serious health and environmental concerns.

Some apparent solutions to reduce high energy neutrino fluxes at large distances are: (A) placing the ring deep underground, (B) building the muon collider at an isolated site, (C) adding a moderate magnetic field over all components and (D) adding a systematic time-varying vertical wave field in the ring.

Detector Design: Beam-Induced Background (BIB) A significant challenge for detector design at a muon collider is managing the beam-induced background (BIB), which refers to the flux caused by decay products [11]. The intense BIB needs to be successfully distinguished from the products of actual $\mu^+\mu^-$ collisions. The advances in detector technologies, spurred by developments for the High-Luminosity LHC upgrade, indicate that high-quality physics research is feasible even in the presence of significant BIB. These technological advancements will be crucial in designing detectors that can isolate true collision events from background noise effectively.

4 Summerize

Building a muon collider represents a transformative step for the high-energy physics community, offering unprecedented opportunities to explore particle physics at energy scales and precision levels beyond the reach of current collider technology. This facility would enable detailed investigations into the properties of the Higgs boson, the potential discovery of new particles predicted by theories beyond the Standard Model, and enhanced studies of dark matter and other fundamental phenomena. The ability of muon colliders to operate at significantly higher energies without substantial synchrotron radiation losses, and with a cleaner experimental environment compared to hadron colliders, positions them as a powerful tool for advancing our understanding of the universe.

However, the development of a muon collider also poses substantial challenges, particularly in the realms of research and development. The primary hurdles include perfecting muon beam cooling techniques to achieve the necessary beam emittance, developing stable and efficient acceleration methods, and managing high radiation levels and intense neutrino fluxes that could pose environmental and safety risks. Additionally, the beam-induced background presents significant difficulties for detector design, requiring sophisticated technologies to effectively distinguish between collision events and background noise. Overcoming these challenges necessitates a concerted international effort, substantial investment in R & D, and breakthroughs in accelerator and detector technologies.

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